

$$\begin{array}{l} t \\ T \\ (0 \leq \\ t \leq \\ T) \\ v(S,t) \\ \{ - \\ \frac{\partial v}{\partial t} + \\ rS \frac{\partial v}{\partial S} + \\ \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 v}{\partial S^2} - \\ \rho v = \\ 0, \\ v(S,0) = \\ \max(S - \\ K,0) 1_{[L,U]}(S), \\ v(S,t) \rightarrow \\ 0 \text{ as } t \rightarrow \\ \infty, \\ 1_{[L,U]}(S) \\ [L,U](S) = \\ \begin{cases} 1 & S \in \\ [L,U], \\ 0 & S \notin \\ [L,U]. \end{cases} \end{array}$$

$$C(S,T)=SN(d_1)-Ke^{-rT}N(d_2),$$

$$d_1=\frac{\log(S/K)+(r+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$(1) \qquad d_2=\frac{\log(S/K)+(r-\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}=d_1-\sigma\sqrt{T}.$$

$$(2) \qquad N(\cdot) \\ N(d_1)=\int_{-\infty}^{d_1}f(u)du=\int_{-\infty}^{d_1}\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}du,$$

$$\begin{array}{l} N'(d_1) \\ N'(d_2) \\ N'(d_1)=\frac{\partial N(d_1)}{\partial d_1}=\frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}, \end{array}$$

$$\begin{array}{l} '(d_2)= \\ \frac{\partial N(d_2)}{\partial d_2}= \\ \frac{1}{\sqrt{2\pi}}e^{-\frac{d_2^2}{2}}= \\ \frac{1}{\sqrt{2\pi}}e^{-\frac{(d_1-\sigma\sqrt{T})^2}{2}}= \\ \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}. \\ e^{d_1\sigma\sqrt{T}}. \\ e^{-\frac{\sigma^2T}{2}} \\ \underline{e}^{-\frac{d_1^2}{2}}. \\ \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}. \\ e^{\log\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)T}. \\ e^{-\frac{\sigma^2T}{2}}= \\ \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}. \end{array}$$

$$\begin{array}{l} \frac{S}{K}. \\ e^{rT}. \\ \S\S\S. \\ (\dot{S}_j,t_n) \\ V_j^n= \\ V_j(S_j,t_n) \\ S \\ S_{\max} \\ [0,S_{\max}]\times \\ [0,T] \\ \Delta S \\ \Delta_t S \\ S_j \\ t_n \\ j= \\ 0,\dots,M \\ S_j= \\ j\Delta S \end{array}$$

$$\left(-\frac{\Delta t}{2}\left(\frac{\sigma S_j}{\Delta S}\right)^2-\frac{\Delta t}{2}\frac{rS_j}{\Delta S}\right)V_{j+1}^{n+1}=$$

$$V_j^n.$$

$$\sigma^2\ll$$

$$\frac{\Delta S}{0.05}=$$

$$\frac{\Delta t}{10^{-4}}=$$

$$\frac{L}{90}=$$

$$\frac{K}{100}=$$

$$\frac{U}{110}=$$

$$\frac{r}{0.05}=$$

$$\frac{t}{0.001}=$$

$$S^{max}=$$

$$\frac{\partial v}{\partial t}$$

$$\frac{\partial v}{\partial S}$$

$$\frac{\partial^2 v}{\partial S^2}$$

$$\frac{\partial^2 v}{\partial S^2}\simeq \frac{1}{2}\left(\frac{V_{j-1}^n-2V_j^n+V_{j+1}^n}{\Delta S^2}+\frac{V_{j-1}^{n+1}-2V_j^{n+1}+V_{j+1}^{n+1}}{\Delta S^2}\right).$$

$$\frac{v}{V_j^{n+1}}$$

$$\frac{n+1}{j}-$$

$$V_j^n\frac{V_{j+1}^{n+1}-V_{j-1}^{n+1}}{\Delta t+rS_j\frac{V_{j+1}^{n+1}-V_{j-1}^{n+1}}{2\Delta S}+\frac{1}{4}\sigma^2S_j^2\left(\frac{V_{j-1}^n-2V_j^n+V_{j+1}^n}{\Delta S^2}+\frac{V_{j-1}^{n+1}-2V_j^{n+1}+V_{j+1}^{n+1}}{\Delta S^2}\right)}=rV_j^{n+1}.$$

$$\left(-\frac{\Delta t}{4}\left(\frac{\sigma S_j}{\Delta S}\right)^2+\frac{\Delta t}{2}\frac{rS_j}{\Delta S}\right)V_{j-1}^{n+1}+$$

$$\left(1+\frac{\Delta t}{2}\left(\frac{\sigma S_j}{\Delta S}\right)^2+r\Delta t\right)V_j^{n+1}$$

$$+$$

$$\left(-\frac{\Delta t}{4}\left(\frac{\sigma S_j}{\Delta S}\right)^2-\frac{\Delta t}{2}\frac{rS_j}{\Delta S}\right)V_{j+1}^{n+1}$$

$$=$$

$$\left(\frac{\Delta t}{4}\left(\frac{\sigma S_j}{\Delta S}\right)^2\right)V_{j-1}^n+$$

$$\left(1-\frac{\Delta t}{2}\left(\frac{\sigma S_j}{\Delta S}\right)^2\right)V_j^n+$$

$$\left(\frac{\Delta t}{4}\left(\frac{\sigma S_j}{\Delta S}\right)^2\right)V_{j+1}^n.$$

$$\sigma^2\ll$$

$$\frac{\Delta S}{0.05}=$$

$$\frac{\Delta t}{10^{-4}}=$$

$$\frac{L}{90}=$$

$$\frac{K}{100}=$$

$$\frac{U}{110}=$$

$$\frac{r}{0.05}=$$

$$\frac{t}{0.001}=$$

$$S^{max}=$$

$$\frac{\partial v}{\partial t}$$

$$\frac{\partial v}{\partial S}$$

$$\frac{\partial^2 v}{\partial S^2}$$

$$V_j^n\frac{V_{j+1}^n-V_{j-1}^n}{\Delta t+rS_j\frac{V_{j+1}^n-V_{j-1}^n}{2\Delta S}+\frac{1}{2}\sigma^2S_j^2\frac{V_{j-1}^n-2V_j^n+V_{j+1}^n}{\Delta S^2}}=rV_j^n.$$

$$\frac{n+1}{j}=$$

$$\frac{j}{j}$$

$$\left(\frac{\Delta t}{2}\left(\frac{\sigma S_j}{\Delta S}\right)^2-\frac{\Delta t}{2}\frac{rS_j}{\Delta S}\right)V_{j-1}^n+$$

$$\left(1-\Delta t\left(\frac{\sigma S_j}{\Delta S}\right)^2-r\Delta t\right)V_j^n$$

$$+$$

$$\left(\frac{\Delta t}{2}\left(\frac{\sigma S_j}{\Delta S}\right)^2+\frac{\Delta t}{2}\frac{rS_j}{\Delta S}\right)V_{j+1}^n.$$